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A new approach to motion control of torque-constrained manipulators by using time-scaling of reference trajectories †

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Abstract

We introduce a control scheme based on using a trajectory tracking controller and an algorithm for on-line time-scaling of the reference trajectories. The reference trajectories are time-scaled according to the measured tracking errors and the detected torque/acceleration saturation. Experiments are presented to illustrate the advantages of the proposed approach.

***Keywords*:** Motion control; Robot manipulator; Constrained torques; Time-scaling; Stability

1. Introduction

In practice, robot motion is specified according to physical constraints, such as limited torque input. Thus, the computation of the desired trajectory is constrained to attain the physical limit of actuator saturations (2). See, e.g., [1], for a review of trajectory planning algorithms considering the robot model parameters and torque saturation. Besides, trajectories can also be planned irrespectively of the estimated robot model, i.e., simply by using constraints of position, velocity and acceleration at each time instant, see, e.g., [2, 3].

Once the reference trajectory is specified, the task execution is achieved in real time by using a trajectory tracking controller. Notwithstanding, if parametric errors in the model estimation are present, and considering that the desired trajectories require to use the maximum torque, no margin to suppress the tracking error will be available. As a consequence, the manipulator deviates from the desired trajectory and poor execution of the task is obtained. On-line time-scaling of trajectories has been studied in the literature as an alternative to solve the problem of trajectory tracking control considering constrained torques and model uncertainties.

A technique for time-scaling of off-line planned trajectories is introduced by Hollerbach [4]. The method provides a way to determine whether a planned trajectory is dynamically realizable given actuator torque limits, and a mode to bring it to one realizable. However, this method assumes that the robot dynamics is perfectly known and robustness issues were not considered. It is noteworthy that this approach has been extended to the cases of multiple robots in cooperative tasks [5] and robot manipulators with elastic joints [6]. To tackle the drawback of the assumption that the robot model in exactly known, Dahl and Nielsen [7] proposed a control algorithm that results in the tracking of a time-scaled trajectory obtained from a specified geometric path and a modified on-line velocity profile. The method considers an internal loop that limits the slope of the path velocity when the torque input is saturated. Other solutions have been proposed in, e.g., Arai *et al.* [8], Eom *et al.* [9], Niu and Tomizuka [10].

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In this paper, an algorithm for tracking control of manipulators under the practical situation of limited torques and model uncertainties is introduced. The proposed approach consists of using a trajectory tracking controller and an algorithm to obtain on-line time-scaled reference trajectories. This is achieved through the control of a time-scaling factor, which is motivated by the ideas introduced by Hollerbach [4]. The new method does not require the specification of a velocity profile, as in [7, 8, 11]. Instead, we formulate the problem departing from the specification of a desired path and a desired timing law.

2. Robot model, desired motion description and control problem formulation

2.1 Robot model

The dynamics in joint space of a serial-chain n-link robot manipulator, considering the presence of friction at the robot joints, can be written as [14]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + F_v\dot{q} + f_C(\dot{q}), = \tau \qquad (1)$$

where q is the $n \times 1$ vector of joint displacements, $\dot{q}$ is the $n \times 1$ vector of joint velocities, τ is the $n \times 1$ vector of applied torque inputs, $M(q)$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(q,\dot{q})\dot{q}$ is the $n \times 1$ vector of centripetal and Coriolis torques, $g(q)$ is the $n \times 1$ vector of gravitational torques, $F_v \in \mathbb{R}^{n \times n}$ is a diagonal positive definite matrix which contains the viscous friction coefficients of each joint, and $f_C(\dot{q}) \in \mathbb{R}^n$ is the vector of Coulomb friction forces. Let us notice that the viscous and Coulomb friction terms satisfy

$$| f_{vi}\dot{q}_i + f_{Ci}(\dot{q}_i) | \le f_{vi} | \dot{q}_i | + f_{Ci}^M,$$

where f_{Ci}^M and f_{vi} are positive constants, and $i = 1,\ldots,n.$

Let T denote the torque space, defined as

$$T = \{\tau \in \mathbb{R}^n : -\tau_i^{max} < \tau_i < \tau_i^{max}\},\ i = 1,\ldots,n, \qquad (2)$$

with $\tau_i^{max} > 0$ the maximum torque input for the i th joint. It is assumed that

$$f_{Ci}^M + | g_i(q) | < \tau_i^{max} i = 1,\ldots,n. \qquad (3)$$

where $g_i(q)$ is the i th element of the vector $g(q)$. Assumption (3) ensures that any joint position q will be reachable.

2.2 Desired motion description

The desired motions of a robot arm can be formulated in terms of a desired geometric path in the joint space, and an associated timing law. A desired path is given as a function

$$q_d(s) : [s_0, s_f] \to \mathbb{R}^n \qquad (4)$$

where s is called path parameter. It is assumed that the desired path attains

$$\frac{d}{ds} q_d(s) \neq 0.$$

This assumption is required in the problem design of time optimal trajectories [1] and also will be necessary in the results exposed in this paper.

The motion specification is completed by specifying a timing law [4]:

$$s(t) : [t_0, t_f] \to [s_0, s_f], \qquad (5)$$

which is a strictly increasing function. The timing law (5) is the path parameter $s(t)$ as a time function. As practical matter, it is possible to assume that the path parameter $s(t)$ is a piecewise twice differentiable function. The first derivative $\dot{s}(t)$ is called *path velocity* and the second derivative $\ddot{s}(t)$ is called *path acceleration* [7].

An important assumption required in the proposed algorithm is that the desired path $q_d(s)$ and the timing law should specified so that

$$s_f > s_0 > 0. \qquad (6)$$

Let us notice that there is a large class of tasks that can be encoded in this way, for instance, periodic and point-to-point motions, which will be shown latter.

A nominal trajectory is obtained from the path $q_d(s)$ and the timing law $s(t)$:

$$q_r(t) = q_d(s(t)). \qquad (7)$$

2.3 Control problem formulation

Let us suppose that only estimations of the robot model (1) are available, namely $\widehat{M}(q)$ for the inertia matrix, $\widehat{C}(q,\dot{q})$ for the centripetal and Coriolis matrix, $\widehat{g}(q)$ for the vector of gravitational forces, $\widehat{F}_v$ for the matrix of viscous friction coefficients, and $\widehat{f}_C(\dot{q})$ for the vector of Coulomb friction forces.

The control problem is to design an algorithm so that forward motion along the path $q_d(\sigma(t))$, with $\sigma(t)$ a strictly increasing function, is obtained as precise as possible:

$$\| q_d(\sigma(t)) - q(t) \| \le \varepsilon, \ \forall\, t \ge t_0, \tag{8}$$

where ε is a positive constant, and considering that the input torque τ must be into the admissible torque space: $\tau \in T$.

It is important to remark that in our approach is not necessary that the specified nominal trajectories (7), which are encoded indirectly by the desired path $q_d(s)$ and the timing law $s(t)$, evaluated for the estimated robot model produce torque into the admissible torque space T in (2). In other words, the following assumption is not necessary:

$$\widehat{M}(q_r)\ddot{q}_r + \widehat{C}(q_r,\dot{q}_r)\dot{q}_r + \widehat{g}(q_r) + \widehat{F}_v\dot{q}_r + \widehat{f}_C(\dot{q}_r) = \tau \in T.$$

This is because the proposed algorithm generates on-line new trajectories that produce admissible control inputs.

3. Tracking control of nominal trajectories

Trajectory tracking control is the classical approach to solve the motions tasks specified by the nominal trajectory (7). Since we have assumed partial knowledge of the robot model, a controller that guarantees robust tracking of the nominal trajectory $q_r(t) = q_d(s(t))$ must be used. Many algorithms have been proposed with this aim, between them we have the following computed torque-based controller [13]:

$$\tau = \widehat{M}(q)\ddot{q}_0 + \widehat{C}(q,\dot{q})\dot{q} + \widehat{g}(q) + +\widehat{F}_v\dot{q} + \widehat{f}_C(\dot{q}), \tag{9}$$

where

$$\begin{aligned} q_0 &= \frac{d^2}{dt^2} q_d(s) + K_v\dot{e}_p + K_p e_p, \quad (10) \\ e_p(t) &= q_d(s(t)) - q(t), \\ \dot{e}_p(t) &= \dot{q}_d(s(t)) - \dot{q}(t), \\ \frac{d^2}{dt^2} q_d(s) &= \frac{\partial^2 q_d(s)}{\partial s^2}\dot{s}^2 + \frac{\partial q_d(s)}{\partial s}\ddot{s}, \end{aligned}$$

and K_p, K_v are $n \times n$ symmetric positive definite matrices.

Considering unconstrained torque input, the control law (9) guarantees that the signals $e_p(t)$ and $\dot{e}_p(t)$ are uniformly ultimately bounded (see, e.g., Lewis *et al.* [9]). Fig. 1 shows a block diagram of implementation of the trajectory tracking controller (9), having as inputs the path parameter $s(t)$, the path velocity $\dot{s}(t)$, and the path acceleration $\ddot{s}(t)$. With respect to the performance requirement (8), in the case of the classical trajectory tracking control, we have that $\sigma(t) = s(t)$.

Notwithstanding, the torque capability of the robot actuators is limited, and frequently desired trajectories are planned using this fact. Thus, room does not exist for extra torque of the feedback control action for compensating the model disturbances, hence a poor tracking performance (8) is presented.

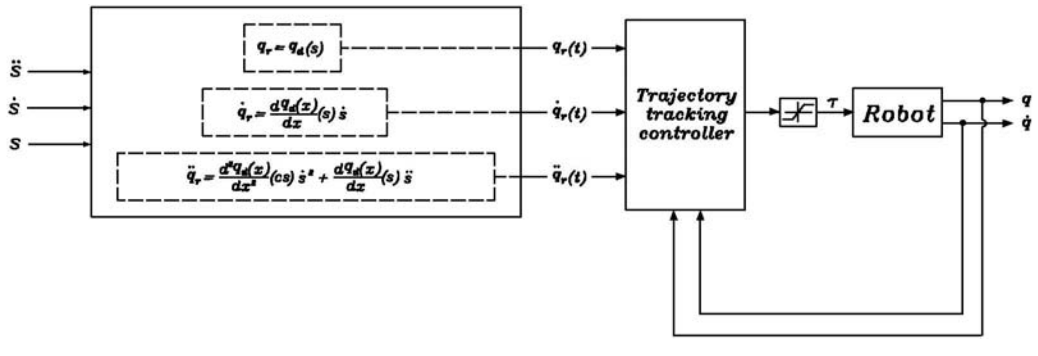

Fig. 1. Block diagram of the classical approach of tracking control of nominal trajectories.

4. Tracking control of on-line time-scaled reference trajectories

Let us define a new path parameter σ in the following way:

$$\sigma(t) = c(t)s(t), \quad (11)$$

where $c(t)$ is a scalar function called *time-scaling factor*, and $s(t)$ is the path parameter given as a time function in an explicit way. The desired trajectory is obtained using the new path parameter (11) into the path (4) as a function of time:

$$\bar{q}_r(t) = q_d(\sigma(t)) = q_d(c(t)s(t)). \quad (12)$$

The signal $c(t)$ in (12) is a time-scaling factor of the nominal trajectory $\bar{q}_r(t)$ in (12). The introduction of a time-scaling factor $c(t)$ is also used in the non robust algorithm proposed in [4]. Let us notice that the nominal, desired, value of the time scaling factor $c(t)$ is one. It is easy to see that when time-scaling is not present, the time evolution of $\sigma(t)$ is identical to $s(t)$, and $\bar{q}_r(t) = q_r(t)$, where $q_r(t)$ is given by (7). An important point in the definition of the time-scaling factor $c(t)$ is that if $c(t) > 1$ the movement is sped up and if $c(t) < 1$ the movement is slowed down. Thus, movement speed can be dynamically changed to compensate errors in the resulting tracking performance of trajectory (12).

The time derivative of $\sigma(t)$ in (11) is given as follows:

$$\dot{\sigma} = \dot{c}s + c\dot{s}.$$

We call the signal $\dot{c}(t)$ *time-scaling velocity* of the nominal trajectory. Further, the path acceleration $\ddot{\sigma}$ is given by

$$\ddot{\sigma} = \ddot{c}s + \gamma, \quad (13)$$

where

$$\gamma = 2\dot{c}\dot{s} + c\ddot{s}. \quad (14)$$

The signal $\ddot{c}(t)$ in Eq. (13) is called *time-scaling acceleration*.

Let us consider the trajectory tracking controller given by

$$\tau = \widehat{M}(q)\ddot{q}_0 + \widehat{C}(q,\dot{q})\dot{q} + \widehat{g}(q) + \widehat{F}_v\dot{q} + \widehat{f}_C(\dot{q}), \quad (15)$$

where

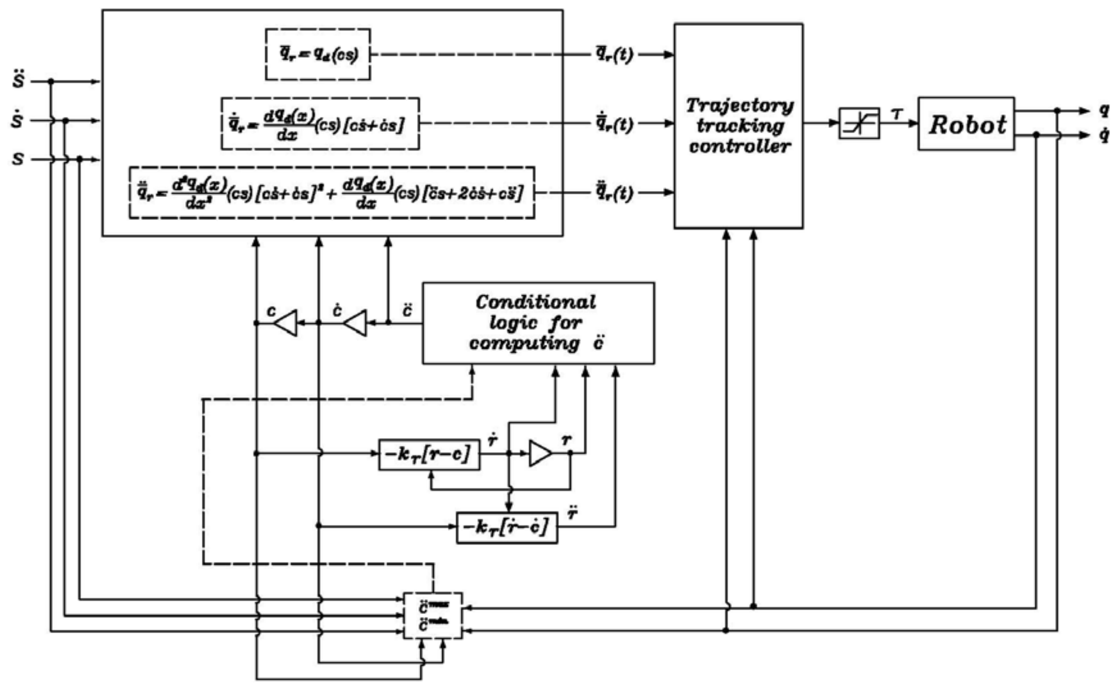

Fig. 2. Block diagram of the proposed approach of tracking control of on-line time-scaled reference trajectories.

$$\ddot{q}_0 = \frac{d^2}{dt^2} q_d(cs) + K_v \dot{\bar{e}}_p + K_p \bar{e}_p, \tag{16}$$
$$\bar{e}_p(t) = \bar{q}_d(c(t)s(t)) - q(t),$$
$$\dot{\bar{e}}_p(t) = \dot{\bar{q}}_r(c(t)s(t)) - \dot{q}(t),$$
$$\frac{d^2}{dt^2} q_d(cs) = \frac{\partial^2 q_d(cs)}{\partial cs^2} \underbrace{[\dot{c}s + c\dot{s}]^2}_{\dot{\sigma}^2} + \frac{\partial q_d(cs)}{\partial cs} \underbrace{[\ddot{c}s + \gamma]}_{\ddot{\sigma}},$$

and K_p, K_v are $n \times n$ symmetric positive definite matrices. Thus, under the control scheme (15) we have that performance requirement (8) is given with $\sigma(t) = c(t)s(t)$

It is worth noting that controller (15) can be written in a parametric form by factoring the time scaling acceleration $\ddot{c}$:

$$\tau = \beta_1(c,s,q)\ddot{c} + \beta_2(c,\dot{c},s,\dot{s},q,\dot{q}), \tag{17}$$

where

$$\beta_1(c,s,q) = \widehat{M}(q) \frac{\partial q_d(cs)}{\partial cs} s, \tag{18}$$
$$\beta_2(\sigma,\dot{\sigma},q,\dot{q}) = \widehat{M}(q) \frac{\partial q_d(cs)}{\partial cs} \gamma$$
$$+\widehat{M}(q) \left[\frac{\partial^2 q_d(cs)}{\partial cs^2} [\dot{c}s + c\dot{s}]^2 + \right.$$
$$\left. + K_v \dot{e}_p + K_p e_p \right]$$
$$+ \widehat{C}(q,\dot{q})\dot{q} + \widehat{g}(q) + \widehat{f}(\dot{q}), \tag{19}$$

and γ is defined in (14).

The path acceleration $\ddot{\sigma}$ in Eq. (13) is expressed in terms of the time-scaling factor $c(t)$, the parameter $s(t)$, and successive time derivatives. In this way, instead of constraining the path acceleration $\ddot{\sigma}(t)$, as done in [7-11], for enforcing the control torque into the admissible limits, it is possible to constrain the scaling acceleration $\ddot{c}(t)$ using the limits of torque input for each joint i, that is,

$$\ddot{c}_i^{min} \le \ddot{c} \le \ddot{c}_i^{max}.$$

The signals $\ddot{c}_i^{min}$ and $\ddot{c}_i^{max}$ are computed in the following way:

$$\ddot{c}_i^{max} = \begin{cases} [\tau_i^{max} - \beta_{2i}]/\beta_{1i}, & \text{for} \quad \beta_{1i} > 0, \\ [-\tau_i^{max} - \beta_{2i}]/\beta_{1i}, & \text{for} \quad \beta_{1i} < 0, \\ \infty, & \text{for} \quad \beta_{1i} = 0, \end{cases}$$

$$\ddot{c}_i^{min} = \begin{cases} [-\tau_i^{max} - \beta_{2i}]/\beta_{1i}, & \text{for} \quad \beta_{1i} > 0, \\ [\tau_i^{max} - \beta_{2i}]/\beta_{1i}, & \text{for} \quad \beta_{1i} < 0, \\ -\infty, & \text{for} \quad \beta_{1i} = 0, \end{cases}$$

where β_{1i} and β_{2i} are given explicitly in Eqs. (18), and (19), respectively.

As previously mentioned in Section 2.2, the path parameter $s(t)$ should be specified with $s_0 > 0$. This restriction avoids that

$$\beta_{1i} = 0, \forall\, i = 1,\ldots,n,$$

which implies that the values of $\ddot{c}_i^{min}$ and $\ddot{c}_i^{max}$ become undetermined. The limits of the time scaling acceleration $\ddot{c}$, which depends on s, $\dot{s}$, $\ddot{s}$, c, $\dot{c}$, and measured signals $q_i, \dot{q}_i$, are given by

$$\ddot{c}^{max} = \min_i \left\{ \ddot{c}_i^{max} \right\} \tag{20}$$
$$\ddot{c}^{min} = \max_i \left\{ \ddot{c}_i^{min} \right\} \tag{21}$$

The limits (20)-(21) provide a way of modifying the reference trajectory (12), via alteration of $\ddot{c}(t)$, so that the torque limits are satisfied. If the resulting nominal reference trajectory $\bar{q}_r(c(t)s(t))$ is inadmissible in the sense of unacceptable torques, then it will be modified by limiting the time scaling acceleration $\ddot{c}(t)$ to satisfy $\tau_i \in T$.

Note that when the time-scaling acceleration $\ddot{c}(t)$ is modified, the value of the time-scaling factor $c(t)$ will change. Thus, considering the limits (20) and (21), we must design an internal feedback to drive the time-scaling factor $c(t)$ in a proper way. The proposed internal feedback is given by

$$\frac{d}{dt} c = \dot{c}, \tag{22}$$
$$\frac{d}{dt} \dot{c} = sat(v; \ddot{c}^{min}, \ddot{c}^{max}), \tag{23}$$

with the initial conditions $c(t_0) = 1$, and $\dot{c}(t_0) = 0$, the saturation function

$$sat(v; \ddot{c}^{min}, \ddot{c}^{max}) = \begin{cases} v & \forall \quad \ddot{c}^{min} \le v \le \ddot{c}^{max}, \\ \ddot{c}^{min} & \forall \quad v < \ddot{c}^{min}, \\ \ddot{c}^{max} & \forall \quad v > \ddot{c}^{max}, \end{cases}$$

and v properly designed.

The input torques $\tau \in \mathbb{R}^n$ are kept within the admissible limits by using the internal feedback (22)-(23), which provides a way to scale in time the reference trajectories $q_d(c(t)s(t))$. We adopt the idea of specifying a time varying desired time-scaling factor.

Let us consider

$$v = \ddot{r} + k_{vc}[\dot{r} - \dot{c}] + k_{pc}[r - c], \tag{24}$$

where k_{vc} and k_{pc} strictly positive constants, and $r(t)$ is the time-scaling reference. The control law (24) is used in the internal feedback (22)-(23).

If the generated control torque is admissible, then the Eq. (23) satisfies $\ddot{c} = v$, v defined in (24). Therefore we can write

$$\frac{d^2}{dt^2}[r - c] + k_{vc}\frac{d}{dt}[r - c] + k_{pc}[r - c] = 0.$$

Because k_{vc} and k_{pc} are strictly positive constants, $[r(t) - c(r)] \to 0$ as $t \to \infty$ in an exponential form.

The nominal value of time-scaling reference is $r(t) = 1$. However, if we use $r(t) = 1$ in (24), we have that the time-scaling factor $c(t)$ will be kept close to 1, even if saturation is detected. Hence the improvement in the tracking performance will be marginal.

If saturation is detected, it means that the desired trajectory $q_d(cs)$ should slow down. So, the time-scaling factor $c(t)$ should be less than one. Thus, we use the idea that the time-scaling reference $r(t)$ is computed by an *estimation* of the actual value of the time-scaling factor $c(t)$. If at time $t = t_1$ the signal $c(t)$ is decreased to kept the control torques $\tau \in \mathbb{R}^n$ into the admissible limits, then it would be convenient to use the desired trajectory,

$$q_d(c(t_1)s(t))$$

for all $t \geq t_1$. A way to implement this idea is by the following time-scaling estimator,

$$\dot{r}(t) = -k_r[r - c], \; r(0) = 1, \tag{25}$$

where k_r is a strictly positive constant. The value of $\ddot{r}(t)$ in (24) is computed differentiating (25) with respect to time:

$$\ddot{r}(t) = -k_r[\dot{r} - \dot{c}].$$

The proposed dynamics for $r(t)$ in (25) obeys the error between $c(t)$ and $r(t)$. Thus, if torque saturation is detected, the time-scaling factor $c(t)$ is modified by Eqs. (22)-(23) to make the reference trajectory $q_d(c(t)s(t))$ feasible, i.e., to make the input torque admissible. As pointed out previously, the time-scaling reference $r(t)$ is interpreted as an estimation of the actual time-scaling factor $c(t)$.

Note that the limits $\ddot{c}_i^{max}$ and $\ddot{c}_i^{min}$ depend on measured quatities β_1 and β_2. It can therefore not be guaranteed that $\ddot{c}_i^{min} < \ddot{c}_i^{max}$. The following conditional logic for computing $\ddot{c}$ guarantees stability of the time-scaling factor $c(t)$:

$$\begin{array}{l} if\,(\ddot{c}_i^{max} < \ddot{c}_i^{min}) \\ \quad \{ \quad if\,(c_i^{min} < v) \\ \quad \ddot{c} = c_i^{min}; \\ \quad else \\ \quad \ddot{c} = v; \quad \} \\ else \\ \quad \{\, \ddot{c} = \max(\min(v, \ddot{c}_i^{max}), \ddot{c}_i^{min}); \quad \} \end{array} \tag{26}$$

Fig. 2 depicts a block diagram of the controller and time-scaling algorithm (17), (22)-(23), (24) and (26).

It is worth noting that the proposed time-scaling method is independent of the definition of path parameter $s(t)$, Thus, several definitions of the path parameter $s(t)$ considering different characteristics in its time evolution can be used along the lines of the proposed time-scaling algorithm. See, e.g, [2, 3], for planning algorithms of the timing law $s(t)$.

5. Experimental results

A planar two degrees-of-freedom direct-drive arm has been built at the CITEDI-IPN Research Center. See Fig. 3 for a CAD drawing and picture of the experimental robot arm. The system is composed by two DC *Pittman* motors operated in current mode with two *Advanced Motion Controls* servo amplifiers. A *Sensoray 626* I/O card is used to read encoder signals with quadrature included and control commands are transferred through the D/A channels. The control system is running in real-time with a 1 kHz sampling rate on a PC over *Windows XP* using *Matlab* with *Simulink* and the *Real-Time Windows Target*.

Since motors are operated in by servo amplifier in

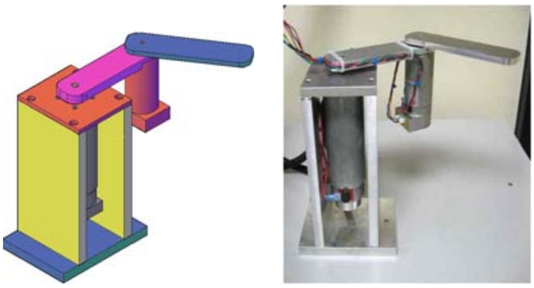

Fig. 3. Experimental robot manipulator actuated by DC motors.

current mode, the output torque of the DC motors is given by

$$\tau_i(t) = k_{mi} I_{mi}(t), \; i = 1,2,$$

where k_{mi} is the motor constant and the $I_{mi}(t)$ is the armature current for the i-th motor. By using a servo applier operated in current mode, we can consider that the actual current is equal to a constant times the input voltage,

$$I_{mi}(t) = k_{ai} u_i(t), \; i = 1,2,$$

with k_{ai} the servo amplifier gain (which is used-defined) and $u_i(t)$ the servo amplifier input voltage. Therefore, the DC motor output torque is

$$\tau_i(t) = k_i u_i(t).$$

with $k_i = k_{mi} k_{ai}$. In this situation, the idea that DC motor actuators can be modeled as ideal torque sources is valid.

Considering the previous discussion, the torque applied to the robot dynamics (1) is given

$$\tau = Ku \tag{27}$$

where K is a 2×2 diagonal positive definite matrix, which for simplicity will be assumed to contain the motor constant of each DC motor actuator, and $u \in \mathbb{R}^2$ is the vector of the servo amplifier input voltages, which at the same time is the control input of the robot dynamics (1).

By virtue of the previous discussion, and from the horizontal configuration implying $g(q) = 0$, the experimental robot arm model can be written as

$$K^{-1}\left[M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F_v \dot{q} + f_C(\dot{q})\right] = u. \tag{28}$$

Previous experiments on identification showed unsymmetric behavior of Coulomb friction force $f_C(\dot{q}) \in \mathbb{R}^2$, which will be defined later. For the sake of simplicity, the model (28) is rewritten as

$$\bar{M}(q)\ddot{q} + \bar{C}(q,\dot{q})\dot{q} + \bar{F}_v \dot{q} + \bar{f}_C(\dot{q}) = u, \tag{29}$$

where the "bar" denotes multiplication times K^{-1}.

Specifically, the entries of the robot model (29) are given as follows:

$$\bar{M}(q) = \begin{bmatrix} \theta_1 + 2\theta_2 \cos(q_2) & \theta_3 + \theta_2 \cos(q_2) \\ \theta_4 + \theta_5 \cos(q_2) & \theta_6 \end{bmatrix} \tag{30}$$

$$\bar{C}(q,\dot{q}) = \begin{bmatrix} -\theta_2 \sin(q_2)\dot{q}_2 & -\theta_2 \sin(q_2)[\dot{q}_1 + \dot{q}_2] \\ \theta_5 \sin(q_2)\dot{q}_1 & 0 \end{bmatrix} \tag{31}$$

$$\bar{F}_v = diag\{\theta_7, \theta_8\}, \tag{32}$$

$$\bar{f}_C(\dot{q}) = \begin{bmatrix} k_1^{-1} f_{C1}(\dot{q}_1) = \begin{cases} \theta_9 \text{ if } \dot{q}_1 \geq 0, \\ -\theta_{10} \text{ if } \dot{q}_1 < 0, \end{cases} \\ k_2^{-1} f_{C2}(\dot{q}_2) = \begin{cases} \theta_{11} \text{ if } \dot{q}_2 \geq 0, \\ -\theta_{12} \text{ if } \dot{q}_2 < 0, \end{cases} \end{bmatrix}. \tag{33}$$

The vector $\bar{f}_C(\dot{q})$ in (33) represents the unsymmetric Coulomb friction term.

It is possible to show that the robot model (29), with entries (30)-(33), can be written in the regression form

$$u = \Omega(q,\dot{q},\ddot{q})\theta$$

where $\Omega(q,\dot{q},\ddot{q}) \in \mathbb{R}^{2\times 12}$ is the regression matrix and $\theta \in \mathbb{R}^{12}$ is the vector of the robot parameters. By using the weighted least squares identification method [14], we have obtained an estimation $\hat{\theta} \in \mathbb{R}^{12}$ of the coefficients involved in robot model (30)-(33). The numerical value of $\hat{\theta} \in \mathbb{R}^{12}$ is shown in Table 1.

5.1 Implementation of the controllers

We have obtained a robot model that is described in the voltage space. Thus, it is convenient to discuss the implementation of controllers (9) and (15) in this situation. The experimental evaluation has the aim of showing the performance of the controllers of the classical trajectory tracking control and the new algorithm, which is based on on-line time-scaling of reference trajectories, taking into account that the applied voltage $u \in V$, where

Table 1. Estimated parameters of the experimental robot arm; see Eqs. (30)-(33) for reference.

Parameter	Value	Unit
$\hat{\theta}_1$	0.0480	Kg m Volt/[N_{rad}]
$\hat{\theta}_2$	0.0037	Kg m Volt/[N_{rad}]
$\hat{\theta}_3$	0.0033	Kg m Volt/[N_{rad}]
$\hat{\theta}_4$	0.0161	Kg m Volt/[N_{rad}]
$\hat{\theta}_5$	0.0220	Kg m Volt/[N_{rad}]
$\hat{\theta}_6$	0.0162	Kg m Volt/[N_{rad}]
$\hat{\theta}_7$	0.0070	Volt-sec/rad
$\hat{\theta}_8$	0.0071	Volt-sec/rad
$\hat{\theta}_9$	0.0571	Volt
$\hat{\theta}_{10}$	0.0067	Volt
$\hat{\theta}_{11}$	0.0554	Volt
$\hat{\theta}_{12}$	0.0105	Volt

$$V = \{u \in \mathbb{R}^2 : -u_i^{max} < u_i < u_i^{max}\},\ i = 1,2, \tag{34}$$

is allowed voltage space.

By using the robot model entries (30)-(33) and the numerical parameters $\hat{\theta} \in \mathbb{R}^{12}$ in Table 1, the classical trajectory tracking controller (9) can be implemented in the voltage space as

$$u = \hat{\hat{M}}(q)\ddot{q}_0 + \hat{\hat{C}}(q,\dot{q})\dot{q} + \hat{\hat{F}}_v\dot{q} + \hat{\hat{f}}_C(\dot{q}), \tag{35}$$

$$\ddot{q}_0 = \frac{d^2}{dt^2}q_d(s) + K_v\dot{e}_p + K_p e_p. \tag{36}$$

The description of the signals involved in the tracking controller (35)-(36) is given in Section 3.

On the other hand, the new time-scaling-based controller (15) is implemented as

$$u = \hat{\hat{M}}(q)\ddot{q}_0 + \hat{\hat{C}}(q,\dot{q})\dot{q} + \hat{\hat{F}}_v\dot{q} + \hat{\hat{f}}_C(\dot{q}), \tag{37}$$

$$\ddot{q}_0 = \frac{d^2}{dt^2}q_d(cs) + K_v\dot{\bar{e}}_p + K_p\bar{e}_p. \tag{38}$$

By following the discussion in Section 4, it is clear that the controller (37)-(38) can be parameterized in the form (17). Thus, the controller (37)-(38) is implemented by using the limits of the time scaling acceleration $\ddot{c}$ in (23) with voltage limit u_i^{max} instead of torque limit τ_i^{max}.

5.2 Results using a circular path

5.2.1 Desired path and timing law

The requested task is to drive the arm in such a way that its joint position $q(t)$ traces a desired path with prescribed velocity in the joint configuration space. The path should be traced with nominal constant tangent velocity. The desired path is

$$q_d(s) = \begin{bmatrix} r_0\cos(v_0[s-1]) \\ r_0\sin(v_0[s-1]) \end{bmatrix},$$

where r_0=1 [rad] and v_0=7.5 [rad/sec]. The timing law, i.e., the path parameter $s(t)$ as function of time, is specified as follows:

$$s(t) = \begin{cases} \frac{1}{2}t^2 + 1, & \text{for } t < 1, \\ [t-0.5]+1 & \text{for } t \geq 1, \end{cases} \tag{39}$$

which is piecewise twice differentiable. Note that the timing law $s(t)$ in (39) achieves the assumption $s_0 > 0$ in (6). In the experiment, the initial configuration of the arm is $q_1(0) = 1$ [rad], $q_2(0)$=0 [rad], and $\dot{q}_1(0) = \dot{q}_1(0) = 0$ [rad/sec]. The experimental tests were run during 10 [sec].

We have assumed that the voltage limits are

$$u_1^{max} = 2 \text{ and } u_2^{max} = 2\,[Volt]. \tag{40}$$

5.2.2 Tracking control of nominal trajectories

To compare the performance of the proposed algorithm, we carried out an experimental test with the trajectory tracking controller given by (35)-(36). The used gains were

$$K_p = diag\{70,70\}\,[1/sec^2], \tag{41}$$

$$K_v = diag\{3.5,3.5\}\,[1/sec]. \tag{42}$$

The experimental results are presented in Fig. 4 which shows the applied voltage; Fig. 5 describes the time evolution of the tracking errors. We note in Fig. 4 that both control voltages are saturated at the same time intervals. This is undesirable, because it produces deviations from the specified desired path, as it can be shown in Fig. 8.

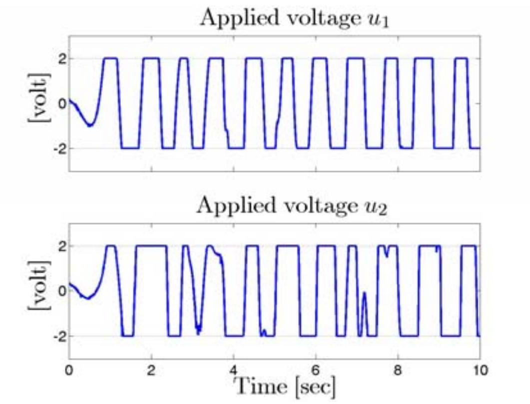

Fig. 4. Tracking control of nominal trajectories: Applied voltage.

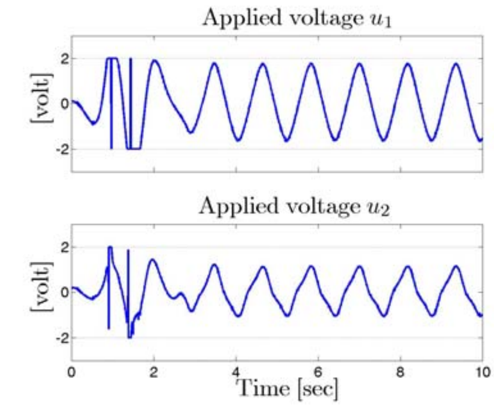

Fig. 6. Tracking control of on-line time-scaled references trajectories: Applied voltages.

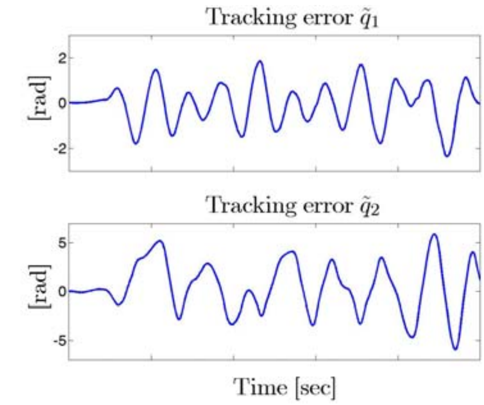

Fig. 5. Tracking control of nominal trajectories: Tracking errors.

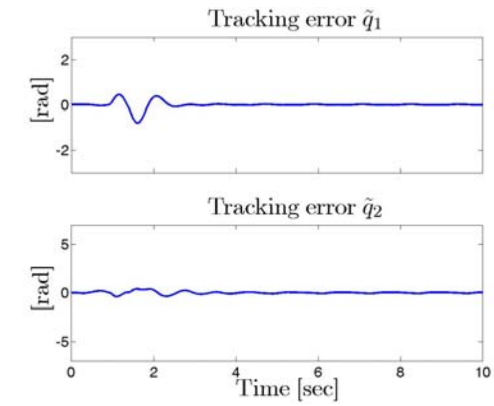

Fig. 7. Tracking control of on-line time-scaled references trajectories: Tracking errors.

5.2.3 Tracking control of on-line time-scaled reference trajectories

We performed an experimental test with the proposed scheme of tracking control of on-line time-scaled reference trajectories (37)-(38) used along with (22), (23), (24)-(25) and the conmutation logic (26). The control gains (41)-(42) were used in the controller (37)-(38). In addition, we used the gains k_{pc} =10 and $k_{vc} = 2\sqrt{k_{pc}}$, in the internal feedback (24), and $k_r = k_{pc}/2$ in the system (25). Fig. 6 shows the applied voltage and Fig. 7 depicts the tracking errors.

5.2.4 Discussions

For the tracking control of on-line time-scaled reference trajectories (37)-(38), it is possible to observe that the applied voltage remains within the admissible limits, and simultaneous saturation does not occur. Moreover, tracking errors are drastically smaller than the tracking errors obtained with tracking control of nominal trajectories (35)-(36). See Fig. 5 and Fig. 7 to compare the performance in the tracking errors of the tracking control of nominal trajectories and the proposed scheme.

With the aim of showing the performance of the controllers tested experimentally, Fig. 8 shows the last 3 seconds of q_2 *vs.* q_1 for the classical trajectory tracking control (35)-(36) and the new algorithm based on time-scaling of reference trajectories (37)-(38). It is clear that the later controller has better performance than the former, since the drawn contour is closer to the desired q_{d2} *vs.* q_{d1}.

Finally, Fig. 9 shows the time evolution of the time-scaling factor $c(t)$, which tends to 0.72; thus the tracking accuracy is improved because the reference trajectory $q_d(c(t)s(t))$ is slowed down.

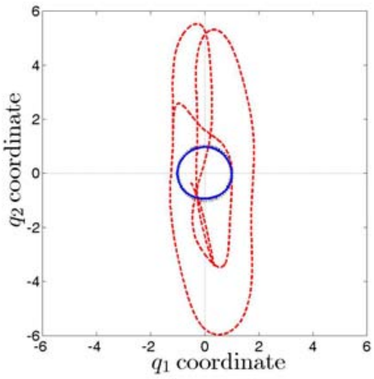

Fig. 8. Comparison of the performed path q_2 *vs*. q_1 : - - - desired path, - - - tracking control of nominal trajectories, --- tracking control of on-line time-scaled references trajectories.

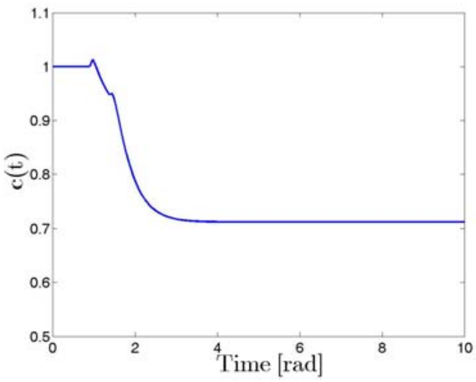

Fig. 9. Tracking control of on-line time-scaled references trajectories: Time-scaling factor.

6. Summary

An approach for trajectory tracking control of manipulators subject to constrained torques has been proposed. A secondary loop to control the time-scaling of the reference trajectory is used, then the torque limits are respected during the real-time operation of the robot.

The new algorithm was experimentally tested in two degrees-of-freedom robot, showing better performance than the classical trajectory tracking controller.

The introduced method is based on rough estimations of the robot model. To increase the robustness, the method would incorporate an adaptive loop to update on-line the estimations of the robot model parameters. However, this is left as further research.

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